Sample Paper 2014: Paper 1

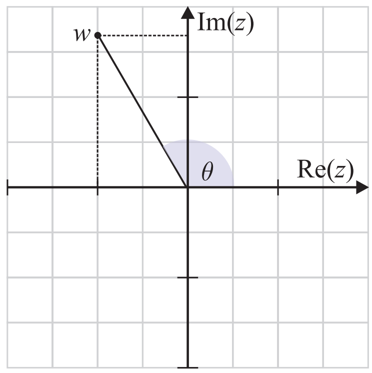
Question 1 (25 marks)

Question 1 (a) (i)

$w = -1 + \sqrt{3}i$

$r = |w| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$|\tan\theta| = \left|\frac{\sqrt{3}}{-1}\right| = \sqrt{3} = \tan\alpha$, where α is the reference angle in the first quadrant.



$\therefore \alpha = \tan^{-1}\sqrt{3} = 60^\circ = \frac{\pi}{3}$

$\therefore \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$w = 2\left[\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right]$ in general polar form.

$\therefore w = 2\left[\cos\left(\frac{2\pi + 6n\pi}{3}\right) + i\sin\left(\frac{2\pi + 6n\pi}{3}\right)\right]$

Question 1 (a) (ii)

Formulae and Tables Book: Algebra (page 20)
De Moivre's theorem
$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

$z^2 = -1 + \sqrt{3}i \Rightarrow z = (-1 + \sqrt{3}i)^{\frac{1}{2}}$

$z = 2^{\frac{1}{2}}\left[\cos\left(\frac{2\pi + 6n\pi}{3}\right) + i\sin\left(\frac{2\pi + 6n\pi}{3}\right)\right]^{\frac{1}{2}}$

$= \sqrt{2}\left[\cos\left(\frac{2\pi + 6n\pi}{6}\right) + i\sin\left(\frac{2\pi + 6n\pi}{6}\right)\right]$

$n = 0: z_1 = \sqrt{2}\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right]$ ← There are 2 roots which you find by putting $n = 0$ and then $n = 1$.

$= \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$n = 1: z_2 = \sqrt{2}\left[\cos\left(\frac{8\pi}{6}\right) + i\sin\left(\frac{8\pi}{6}\right)\right]$

$= \sqrt{2}\left[\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right]$

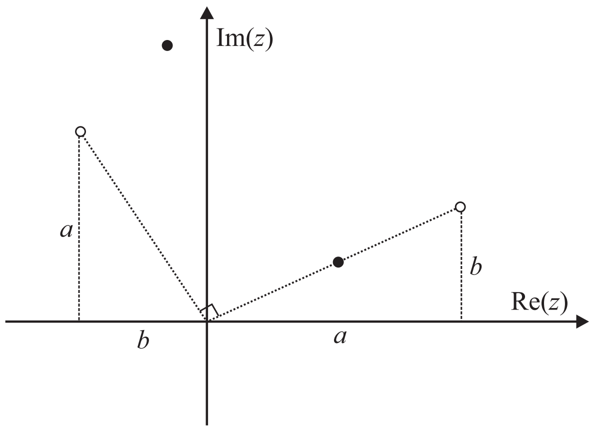
$= \sqrt{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

Question 1 (b)

(i) Let $z_1 = a + bi$.

$z_2 = iz_1 = i(a+bi) = ai + bi^2 = -b + ai$

Notice that the real and imaginary parts are swapped between z_1 and z_2. Can you pick out two points on the diagram where this might be the situation?



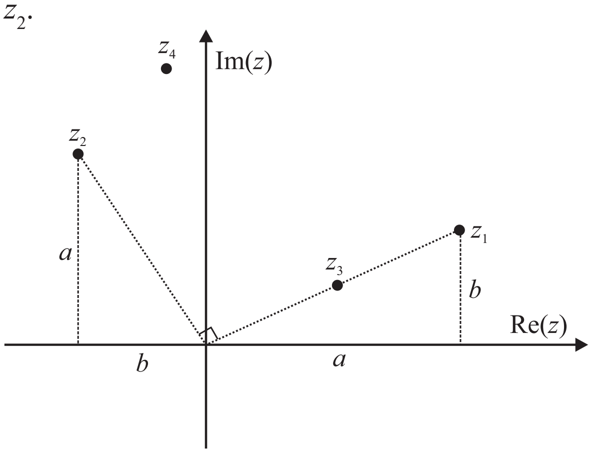
The two hollow points seem to satisfy this condition.
At this stage I don't know which point is z_1 and z_2.

$z_3 = kz_1 = k(a+bi) = ka + kbi$

z_3 and z_1 are in a straight line with the origin.

$z_4 = z_2 + z_3 = (-b+ai) + (ka+kbi)$
$\quad = (ka-b) + (a+kb)i$

You are now in a position to mark in all the points.



(ii) $k \approx \frac{1}{2}$